INDUCTANCE CALCULATION OF A COIL GUN THAT LAUNCHES A THIN PLATE EDGE-ON

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ABSTRACT

A coil gun is a type of induction launcher that induces an eddy current in a metal projectile by a time varying magnetic induction produced by a launch coil. The interaction between the magnetic induction of the coil and the eddy currents can launch the plate with the velocity vector in the plane of the plate, i.e. edge-on[1] A previous published calculation of the eddy current in thin plates[2] includes the conductivity of the plate and the time variation of the applied magnetic field, which complicates the problem. Although these features are necessary for a general treatment of the subject, they complicate the problem and may not be necessary for applications where the skin depth is small compared to the length or width of the plate. For simplification, it has been assumed that the rectangular plate has an infinite conductivity, the plate has zero thickness, the applied magnetic induction is static, and the streamline function for the eddy currents can be expressed as a polynomial with adjustable coefficients. The curl of the stream line function yields the current density distribution. Under these assumptions, the integrals that result from the application of the Biot-Savart law are in principle analytic for any order of the polynomial, but their functional forms are very complicated. Their values, however, can be found by using recursion relations[3]. After the evaluation of these integrals, the coefficients of the polynomial are adjusted so that the boundary conditions for the magnetic induction are satisfied on the plate's surface. This streamline function is then used to find the mutual inductance between the launch coil and plate for the given position. This procedure is repeated for other plate positions. With these results, it is possible to design the power supply for the coil gun and to predict the plate's velocity.

INTRODUCTION

An EM launcher that is well suited for edge-on launching of plates is the reconnection gun, a type of coil gun, invented by Cowan[4]. In this launcher, a time varying magnetic induction produced in an external launch coil induces a current in the plate to be launched. This action is similar to that of a transformer where the primary winding, the external launch coil, induces a current in the secondary winding, the plate. The force between the induced current and the external magnetic induction accelerates the plate out of the coil. Cowan and colleagues[5] demonstrated that high velocities can be achieved by using a number of external coils arranged in line along a path. As a plate was passing through a coil, a current pulse was delivered at the proper time to accelerate the plate toward the next coil. This resulted in the launching of a 150 g aluminum plate to a velocity of 1.0 km/s. Because of the size and weight of the multi-stage gun excludes their use for some applications, single stage launchers are considered here.

SINGLE STAGE COIL GUN

The electrical schematic of a single stage coil gun[6], figure 1, is very similar to a series LRC circuit. Unlike the classic LRC circuit, the inductance associated with the launcher depends on the position of the plate within the coil. This dependence is attributable to the distribution of the magnetic induction in the core of the coil and around the plate. The resistor in this figure represents the energy losses within the system that may also vary with time, but it is usually assumed to have a constant resistance for the purposes of modeling.

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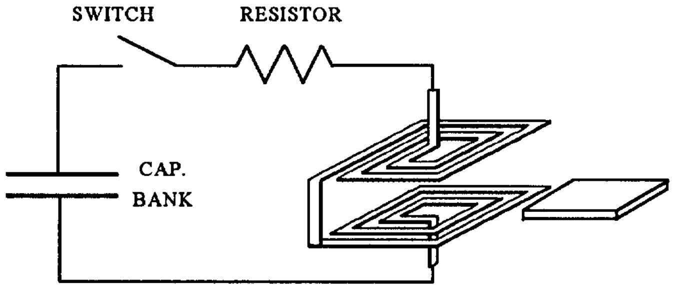

Figure 1. Schematic of a Single Stage Coil Gun

In designing a coil gun for the best performance, it is necessary to calculate the current through the coil, the plate's velocity, and the plate's position in the coil. These calculations require that the inductance of the coil and its gradient be known for any plate position. The coil's inductance is necessary to calculate the coil's current, and the inductance gradient is necessary to calculate the acceleration of the plate. Previously, models of proposed launch coil designs were made from aluminum, and then their inductances were measured with an LRC meter. This procedure was tedious because of the number of coils that had to be constructed and measured. If it were possible to easily calculate the inductance and the gradient of these coils, then a more complete study of various coils with different design parameters and geometries could have been performed.

EDDY CURRENT CALCULATIONS

In a preliminary calculation[7], it was assumed that the eddy current of the plate was distributed only on the surface of the plate with zero skin depth. This eddy current was modeled by covering the top and the bottom of the plate by an array of filamentary rectangular loops. To model the condition in which the magnetic induction produced by the eddy current is equal and opposite to the coil's magnetic induction in the plate, the current in each loop was chosen to make the total magnetic induction zero at the center of each loop. Once the current in each loop was calculated, the force on each loop was found and summed to give the total force on the plate which depends on the inductance gradient through the relation,

$$F(x) = \frac{I^2 L'(x)}{2} \quad , \tag{1}$$

where *F(x)* is the force on the plate, *I* is the current in the coil, and *L'(x)* is the inductance gradient of the coil. This calculation was repeated for a number of plate positions, and the results were compared with the inductance gradient as determined from the inductance meter measurements. This procedure was repeated for a number of plate dimensions, plate materials, and coil designs. The calculated inductance gradient compared favorably with the measured inductance gradient in all cases, provided that the plate was a good conductor and had a thickness larger than the skin depth.

The current loops in the preliminary calculation are now replaced by a continuous current distribution, and it is assumed that the rectangular plate has no thickness. The magnetic induction produced by an eddy current in this thin plate is given by the Biot-Savart law:

$$\vec{B}_e(\vec{x}) = \frac{1}{4\pi\mu_o} \int_{-a}^{a} dx' \int_{-b}^{b} dy' \frac{\vec{J}(x',y') \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} \tag{2}$$

where a is half the length and b is half the height of the plate. If the plate has infinite conductivity, then one must find an eddy current in the plate so that its magnetic induction, $\vec{B}_e(\vec{x})$, is equal and opposite to

the coil's magnetic induction, $\vec{B}_c(\vec{x})$, everywhere on the plate. An approximate current distribution can be found by first assuming that it can be generated from linear combination of the basis functions of the form

$$T(x',y') = (a^2 - x'^2)(b^2 - y'^2)\sum_{i,j=0}^{I,J} T_{i,j} x'^i y'^j \tag{3}$$

where

$$J_x(x',y') = \frac{\partial T(x',y')}{\partial y'} \tag{4}$$

and

$$J_y(x',y') = -\frac{\partial T(x',y')}{\partial x'} \quad . \tag{5}$$

I and J in equation 3 are the maximum powers of x' and y' for the summation. Their exact values depend on the desired degree of approximation. To illustrate how equation 3 generates a physical current distribution, consider a current distribution described by a single term,

$$T_{i,j}(x',y') = (a^2 - x'^2)(b^2 - y'^2)x'^i y'^j T_{i,j} \tag{6}$$

where

$$J_x(x',y') = \frac{\partial T_{i,j}(x',y')}{\partial y'} = T_{i,j}(a^2 - x'^2)x'^i y'^{j-1}(jb^2 - (j+2)y'^2) \tag{7}$$

and

$$J_y(x',y') = -\frac{\partial T_{i,j}(x',y')}{\partial y'} = -T_{i,j}(b^2 - y'^2)x'^{i-1} y'^j (ia^2 - (i+2)x'^2) \quad . \tag{8}$$

This current distribution conserves charge and satisfies the boundary conditions at the edges for all values for i and j. Because each term in equation 3 generates a physical current density distribution, their sum will also generate a physical current density distribution for any choice of coefficients $T_{i,j}$. When equations 7 and 8 are substituted into the Biot-Savart law, equation 3, the magnetic induction in the z-direction is

$$B_z(x,y,z) = \sum_{i,j=0}^{I,J} T_{i,j} I_z(x,y,z)_{i,j} \tag{9}$$

where

$$\begin{aligned} I_z(x,y,z)_{i,j} &= \frac{\mu_o}{4\pi}\int_{-a}^{a} dx' \int_{-b}^{b} dy' \frac{(y-y')(a^2 - x'^2)x'^i y'^{j-1}(jb^2 - (j+2)y'^2)}{\left((x-x')^2 + (y-y')^2 + z^2\right)^{3/2}} \\ &+ \frac{\mu_o}{4\pi}\int_{-a}^{a} dx' \int_{-b}^{b} dy' \frac{(x-x')(b^2 - y'^2)x'^{i-1} y'^j (ia^2 - (i+2)x'^2)}{\left((x-x')^2 + (y-y')^2 + z^2\right)^{3/2}} \end{aligned} \tag{10}$$

All the integrals in equation 10 can be evaluated by using a set of recursive relations. Similar expressions may be written for the x and y components, but they are not needed because they are zero when z = 0. By choosing a set of field points on the plate, it is possible to find values for the coefficients so that the magnetic induction produced by the eddy current, equation 9, is equal and opposite to the coil's magnetic induction at these points. As an example, assuming that I=1 and J=1, there are four unknown coefficients, $T_{0,0}$, $T_{0,1}$, $T_{1,0}$, and $T_{1,1}$, that are determined by four simultaneous equations. These equations result from making $B_z(x,y,0) = -B_c(x,y,0)$ at four points:

$$B_z(x_1,y_1,0) = -B_c(x_1,y_1,0) = T_{0,0}\, I_z(x_1,y_1,0)_{0,0} + T_{0,1}\, I_z(x_1,y_1,0)_{0,1} + T_{1,0}\, I_z(x_1,y_1,0)_{1,0} + T_{1,1}\, I_z(x_1,y_1,0)_{1,1},$$
$$B_z(x_2,y_2,0) = -B_c(x_2,y_2,0) = T_{0,0}\, I_z(x_2,y_2,0)_{0,0} + T_{0,1}\, I_z(x_2,y_2,0)_{0,1} + T_{1,0}\, I_z(x_2,y_2,0)_{1,0} + T_{1,1}\, I_z(x_2,y_2,0)_{1,1},$$
$$B_z(x_3,y_3,0) = -B_c(x_3,y_3,0) = T_{0,0}\, I_z(x_3,y_3,0)_{0,0} + T_{0,1}\, I_z(x_3,y_3,0)_{0,1} + T_{1,0}\, I_z(x_3,y_3,0)_{1,0} + T_{1,1}\, I_z(x_3,y_3,0)_{1,1},$$
$$B_z(x_4,y_4,0) = -B_c(x_4,y_4,0) = T_{0,0}\, I_z(x_4,y_4,0)_{0,0} + T_{0,1}\, I_z(x_4,y_4,0)_{0,1} + T_{1,0}\, I_z(x_4,y_4,0)_{1,0} + T_{1,1}\, I_z(x_4,y_4,0)_{1,1}. \quad (10)$$

The exact positions of the points are arbitrary as long as they are not on the plate's edge. Choosing the points on a uniform grid works well. The resulting set of equations can be solved by using standard methods in linear algebra to find the coefficients. Because these points are fixed on the plate, the integrals do not change when the plate is moved away from the coil. Thus, the matrix, whose elements are the integrals, needs to be evaluated and inverted only once. The components of the vector that must be multiplied by the inverse matrix, however, are negative of the coil's magnetic induction at each point. Because these components depend on the relative position between the coil and the plate, they must be reevaluated when the plate's position is changed.

DISCUSSION

This analysis was applied to a launch coil that was constructed from two plates of a copper-beryllium alloy . Each 15.24x15.24x0.64 cm plate was milled into a square helix by a 0.64 cm diameter end mill. The milling pattern left a conductor with cross-sectional dimensions of 0.64x0.64 cm in a square helix with five complete turns. The square helixes were mounted parallel to each other separated by 5.08 cm. The magnetic induction of the coil at the grid points was estimated by replacing the conductors with a current-carrying filament located at the geometric center of their cross sections. Thus, the magnetic induction of the current-carrying filaments should be approximately the same as the coil, provided that the plate is not too close to a coil's conductor.

In figure 2, the 15x15 cm plate was positioned half way out of the coil. The vertical line through the center of the plate is the location of the outside edge of the launch coil. Thus the portion of the plate to the left of this line is inside the coil, and portion to the right is outside. The lines within the plate are the streamlines for a current density that cancels the coil's magnetic field at 100 grid points, I = 9 and J = 9. The direction of the current density is tangential to the line and the magnitude is inversely proportional to the distance between neighboring lines. The concentration of the streamlines along the left edge of the plate indicates an area where there is a large eddy current and force on the plate. The force accelerates the plate to the right and out of the coil.

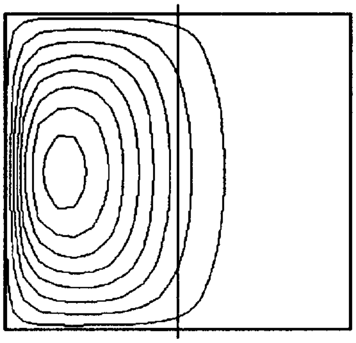

Figure 2: Current streamlines for a plate half way out of the coil.

When the current density is known, the mutual inductance is given by

$$M = \frac{1}{I^2} \int_{-a}^{a} dx \int_{-b}^{b} dy\, B_c(x,y)\, T(x,y) \quad (11)$$

Because the coil's magnetic induction and the eddy currents are proportional to the coil's current *I*, this mutual inductance will be independent of the current. Figure 3 shows the calculated mutual inductance as a solid line along with the measured mutual inductance, the diamonds. The plate position in figure 3 is given as the distance between the centers of the coil and the plate. The consistency between the measured and the calculated mutual inductances indicates that this analysis may be adequate for modeling the performance of a launch coil.

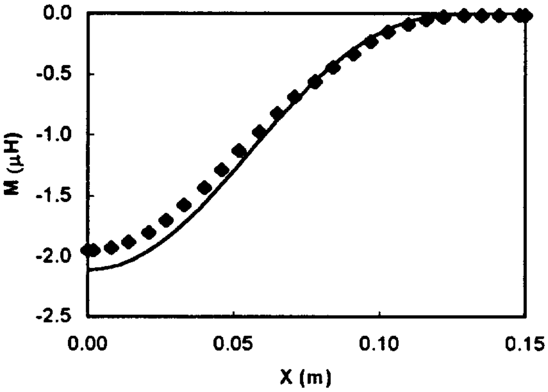

Figure 3. Calculated (Solid Line) and Measured (Diamonds) Mutual Inductance

Eddy currents in a plate were evident when the surface temperature distribution of a plate was observed just after it was launched in an earlier experiment[6]. An aluminum plate was fabricated with a nose section that held a nail oriented along the direction of plate motion. In a low velocity launch, the nail was driven into a plywood barrier by the plate. In this manner, the assembly was captured for viewing by an infrared (IR) video camera (8-12μm) immediately after launch. The aluminum plate had a thick anodized coating to increase the emissivity of the surface for IR radiation. The IR video image in figure 4a shows that the top surface of the plate was heated along its trailing and side edges. By observing the captured plate for some time with the IR video camera, we found that the time for the heat to diffuse from the edges was long compared to the time to launch the plate. Thus, it was concluded that there was negligible heat diffusion during launch. Assuming no heat diffusion and that the conductivity is independent of temperature, the heating rate is then proportional to the square of the magnitude of the current density. The current density was calculated for the plate at its initial position inside the coil, because the plate remained close to the initial position during the entire current pulse for this low velocity shot. The shape of the contour lines, figure 4b, for the heating rates agrees qualitatively with the IR video image. No attempt was made to correlate the heating rate with the observed intensity of the IR radiation.

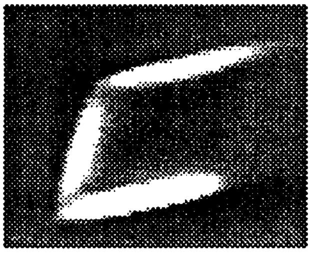

Figure 4a. IR Video Image

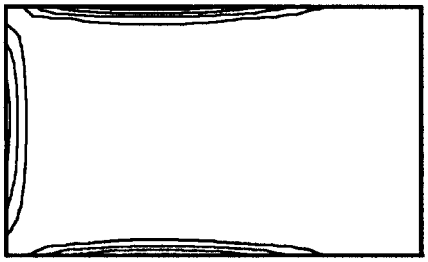

Figure 4b. Heating Rate Contour Lines

The heating at the side edges is caused by the geometry of the box coil. The coil was made from a 10 cm by 10 cm square tube of aluminum 23 cm long with 3 mm thick walls. Its sides were slotted so that the remaining aluminum formed a square helical coil with nine turns. The spacing between one pair of windings was widened in order to permit insertion of the metal plate into its core

from the side. Thus, the proximity of the coil windings along the trailing edge and the side edges of the plate caused large eddy currents and appreciable heating.

CONCLUSION

This calculation of the eddy currents and the mutual inductance depends on three assumptions: (1) the skin depth is zero, (2) the plate has zero thickness and (3) the eddy current density distribution in the plate can be described by a polynomial. The first assumption was made in a preliminary calculation that produced very good agreement with experimental measurements where there is a skin depth. These results indicated that the skin depth may be small compared to some dimension of the plate. After estimating the skin depth and considering the distribution of the magnetic induction around the plate, it was concluded that the skin depth should be compared with the length or the width of the plate and not with its thickness. This calculation also demonstrated that there was a small variation in the inductance gradient when the thickness of the plate was varied, and a plate with no thickness could be assumed. Although these first two assumptions make the problem easier to solve, they may introduce an infinite current density at some of the edges. Strictly speaking, this behavior invalidates the third assumption because a polynomial cannot be infinite at the edges. Despite this complication, it is possible to get meaningful results with some testing and reasonable precautions.

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